pst-bezier

A PSTricks package for drawing Bezier curves; v.0.03

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References
The \texttt{pstricks} package provides (essentially) two main macros for drawing curves: \texttt{\textbackslash pscurve} and \texttt{\textbackslash psbezier}. Both macros employ Bezier splines.

The \texttt{\textbackslash pscurve} macro takes multiple interpolated points as arguments. Thus, it is easy to draw long multiply bent curves. The problem with \texttt{\textbackslash pscurve} is that there is no easy way to change the automatically computed control points without simultaneously changing the interpolated points. Note that some control is possible via the \texttt{curvature} option.

The \texttt{\textbackslash psbcurve} macro gives full control over the interpolation points and the control points of one Bezier polynomial of degree three (two interpolated points and two control points).

Thanks to:
Jean-Côme Charpentier.
1 Introduction

If one demands for the access to certain control points of one multiply bent curve one has to use multiple instances of the \psbezier macro. With this approach each inner interpolation point of the curve has to be input twice. Furthermore, if one needs smooth joints one has to compute control points symmetrically to the corresponding interpolation points for every joint even if one does not care so much about the exact tangential direction at some of those joints. That can be rather tedious.

The \psbcurve macro of the package pst-bezier is intended to demonstrate a way to combine the nice properties of the macros \pscurve and \psbezier. It provides an easy input format to describe ‘arbitrarily’ many interpolation points of a curve and to fix the control points at some freely selected interpolation points.

Note, that pst-bezier is no final package (e.g. the automatical computation of the control points is not as refined as that one for the macro \pscurve).

2 Installation and usage of pst-bezier.tex

Installation: As prerequisites for pst-bezier you need resent working versions of \LaTeX{} and pstricks. The files pst-bezier.tex and pst-bezier.sty must be somewhere in your \TeX{}-input path. Furthermore, the file pst-bezier.pro must be in some path, where dvips can find it.

Usage: As usual, load the packages pstricks and pst-bezier in that order via the \usepackage macro.

Now you are ready to use the \psbcurve macro within your document body. This macro is described in the next section with all its options.

With the following simple \LaTeX{}-source code you can test whether you have correctly installed the package:

```latex
\documentclass{minimal}
\usepackage{pstricks}
\usepackage{pst-bezier}
\begin{document}
\begin{pspicture}(0,-0.4)(6,2)
\psbcurve(1,2)(5,2) % Draw just one straight line.
\end{pspicture}
\end{document}
```

3 The \psbcurve macro

In the most simple form you can specify any number of interpolation points as the argument of \psbcurve.
As usual, options can be specified within brackets.

As you can see in the above example, the showpoints feature works (partially) with \psbcurve. The next figure shows again the curve from the first example. This time labels are added to the points (this is just for the following description, it is not a feature of \psbcurve).

The points labeled with $\vec{p}_k$ ($k = 0, \ldots, 3$) are the interpolation points, these ones labelled with $\vec{l}_1, \ldots, \vec{l}_3$, and these ones labelled with $\vec{r}_1, \ldots, \vec{r}_3$ are the left and right control points, respectively.

Between each consecutive pair $\vec{p}_{k-1}, \vec{p}_k$ of interpolation points the \psbcurve macro draws a cubic Bezier spline. The control points $\vec{l}_k$ and $\vec{r}_k$ determine the tangential direction of the bezier spline at the interpolation points. More exactly, the bezier spline from $\vec{p}_{k-1}$ to $\vec{p}_k$ is tangent to the vector $\vec{l}_k - \vec{p}_{k-1}$ at the point $\vec{p}_{k-1}$ and tangential to the vector $\vec{r}_k - \vec{p}_k$ at the point $\vec{p}_k$. 

\begin{pspicture}[showgrid](0,-0.4)(5,3)
\psbcurve[showpoints](1,1)(2,2)(3,1)(4,2)
\end{pspicture}
Without any optional modifier arguments (described later in this text) the control points are computed automatically from the interpolation points by the formulas\footnote{Note that this method is very crude. To compute the curve such that the curvature is continuous would require solving a nonlinear system of equations. That is not implemented yet.}

\[
\begin{align*}
\vec{l}_1 &= \vec{p}_0 \\
\vec{r}_k &= t_k(\vec{p}_{k-1} - \vec{p}_{k+1}) \quad &\text{for } k = 1, \ldots, n - 1 \\
\vec{r}_n &= \vec{p}_n
\end{align*}
\]

where \( t_k \) \((k = 1, \ldots, n)\) are real coefficients which are called tension and which default to the value \texttt{bcurveTension=}0.25.

You can change the appearance of the curve by several modifiers. First of all you can directly set the left and right control points via the modifiers \texttt{l(x,y)} and \texttt{r(x,y)}, resp., as shown in the next two examples. The unmodified curve is drawn in the background in \texttt{blue} color.

On the right hand side the last example is shown once more without grid and with \texttt{showpoints=false}. There, you see that there is a corner at the second interpolation point.
If you change some left control point \( \vec{l}_k \) with the help of the \( L(x,y) \) modifier then the control point \( \vec{r}_{k-1} \) is set symmetrically to \( \vec{l}_k \) with respect to the interpolation point \( \vec{p}_{k-1} \). In that way you get a smooth joint as demonstrated in the next example.

With the \( t\{t\} \) modifier you can change the tension of the automatically computed control points of the current Bezier spline.

As you can see from the example both control points of the current spline are affected by the \( t\{t\} \) modifier. If you want to change the tension of just the left or right control point you can use the \( tl\{t\} \) or \( tr\{t\} \) modifier, respectively, as demonstrated in the following two examples.
The \texttt{\textbackslash pscurve} macro...

\begin{pspicture}(0,-0.4)(5,3)
\psset{showpoints}
\psbcurve[linecolor=blue,linewidth=0.01](1,1)%(2,2)(3,1)(4,2)
\psbcurve(1,1)(2,2)\texttt{tl}(0.5)(3,1)(4,2)
\endpspicture

The \texttt{ts\{f\}} modifier changes the tension of the left and right control points next to the interpolation point which stands in front of the modifier. In the next example a negative tension value leads to a rather surprising effect.

\begin{pspicture}(0,-0.4)(5,3)
\psset{showpoints}
\psbcurve[linecolor=blue,linewidth=0.01](1,1)%(2,2)(3,1)(4,2)
\psbcurve(1,1)(2,2)\texttt{tr}(0.5)(3,1)(4,2)
\endpspicture

The default value of the tension can be set with the option \texttt{bcurveTension} as in the following example.
The \psbcurve macro

\begin{pspicture}(0,-0.4)(5,3)
\psset{showpoints}
\psbcurve[linecolor=blue,linewidth=0.01](1,1)
%(2,2)(3,1)(4,2)
\psbcurve[bcurveTension=0.5](1,1)
%(2,2)(3,1)(4,2)
\endpspicture

You can set this option also with the help of the \psset macro. It is even possible to change the value of bcurveTension in the middle of a \psbcurve. Just use the modifier $T\{t\}$ for that purpose as shown in the following example.

\begin{pspicture}(0,0.6)(5,6)
\psset{showpoints}
\psbcurve[linecolor=blue,linewidth=0.01](1,1)
%(2,2)(3,1)(4,2)(4,4)(3,5)
%(2,2)(3,1)(4,2)
\psbcurve[bcurveTension=0.5](1,1)
%(2,2)(3,1)(4,2)
\psbcurve[T=0.5](4,4)(3,5)(2,4)(1,5)
\endpspicture

Certainly, you can use the $T\{t\}$ modifier several times in one curve. (Try it for yourself.) The linestyle and fillstyle options (and several more) are respected by \psbcurve as the following example shows.
3.1 Things that do not work (‘known bugs’)

As already mentioned this project is something like an experiment. So, there are many things that do not work.

- new lines inside the argument list are not ignored.
- The control points are computed in a rather crude way (see above). The curvature option is not recognised.
- If fillstyle is set to solid and showpoints then the fill color covers the interpolation and control points.
- arrow heads do not work.
4 Bezier curve with weighted points

4.1 Mathematical background

A mass point is a weighted point \((P; \omega)\) with \(\omega \neq 0\) or a vector \((\overrightarrow{P}; 0)\) with a weight equal to 0. A generic mass point is noted \((P; \omega)\).

Using the quadratic Bernstein polynomials, a rational quadratic Bézier curve having three control mass points \((P_0; \omega_0), (P_1; \omega_1)\) and \((P_2; \omega_2)\), is defined as follow:

**Definition 1 : Rational quadratic Bézier curve (BR curve)**

Let \(\omega_0, \omega_1\) and \(\omega_2\) be three real numbers. Let \((P_0; \omega_0), (P_1; \omega_1)\) and \((P_2; \omega_2)\) be three mass points, these points are not collinear.

Define two sets \(I = \{i | \omega_i \neq 0\}\) and \(J = \{i | \omega_i = 0\}\).

Define the function \(\omega_f\) from \([0; 1]\) to \(\mathbb{R}\) as follows

\[
\omega_f (t) = \sum_{i \in I} \omega_i \times B_i (t)
\]

A mass point \((M; \omega)\) or \((\overrightarrow{u}; 0)\) belongs to the quadratic Bézier curve defined by the three control mass points \((P_0; \omega_0), (P_1; \omega_1)\) and \((P_2; \omega_2)\), if there is a real \(t_0\) in \([0; 1]\) such that:

- if \(\omega_f (t_0) \neq 0\) then we have
  \[
  \overrightarrow{OM} = \frac{1}{\omega_f (t_0)} \left( \sum_{i \in I} \omega_i B_i (t_0) \overrightarrow{O \overrightarrow{P}_i} \right) + \frac{1}{\omega_f (t_0)} \left( \sum_{i \in J} B_i (t_0) \overrightarrow{P}_i \right)
  \]

- if \(\omega_f (t_0) = 0\) then we have
  \[
  \overrightarrow{u} = \sum_{i \in I} \omega_i B_i (t_0) \overrightarrow{O \overrightarrow{P}_i} + \sum_{i \in J} B_i (t_0) \overrightarrow{P}_i
  \]

The reduced discriminant of the denominator \(\omega_f (t_0)\) is

\[
\Delta' = \omega_1^2 - \omega_2 \omega_0
\]

and we can state the following fundamental result:

* if \(\omega_1^2 - \omega_2 \omega_0 = 0\) then the denominator has one and only one root, the curve is a parabolic arc;
* if \(\omega_1^2 - \omega_2 \omega_0 > 0\) then the denominator has two distinct roots, the curve is a hyperbolic arc;
* if \(\omega_1^2 - \omega_2 \omega_0 < 0\) then the denominator does not vanish, the curve is an elliptical arc.

We can note w.l.o.g.\(^2\) that one of the weights can be equal to 1. If \(\omega_0\) is not equal to 0, we choose \(\omega_0 = 1\), else, we choose \(\omega_1 = 1\), and we can characterise the type of the conic from the mass points of the BR curve, see Table 1.

From the access rights used by Unix and Linux, we define a bijection \(f\) between \(\mathbb{F}_2^3 - \{(0, 0, 0)\}\) and the set \(\{1, 2, 3, 4, 5, 6, 7\}\). From \((\omega_2, \omega_1, \omega_0)\), we define a triplet \((b_2, b_1, b_0)\) as follow: if \(w_1 \neq 0\) then \(b_i = 1\) else \(b_i = 0\). Then

\[
f (\omega_2, \omega_1, \omega_0) = b_2 \times 4 + b_1 \times 2 + b_0
\]

\(^2\) We can permute the role of \(P_0\) and \(P_2\)
<table>
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<th>Three weighted points</th>
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<td>Parabola</td>
<td>((P_0; 1), (P_1; \omega)) ((P_2; \omega^2))</td>
<td>((P_0; 1), (\vec{P}_1; 0)) ((\vec{P}_2; 0))</td>
</tr>
<tr>
<td>Ellipse</td>
<td>((P_0; 1), (P_1; \omega_1), (P_2; \omega_2), \omega_2 &gt; \omega_1^2)</td>
<td>((P_0; 1), (\vec{P}_1; 0)) ((P_2; 1))</td>
</tr>
<tr>
<td>Hyperbola</td>
<td>((P_0; 1), (P_1; \omega_1)) ((P_2; \omega_2), \omega_2 &lt; \omega_1^2)</td>
<td>((P_0; 1), (\vec{P}_1; 0)) ((P_2; -1)) ((\vec{P}_0; 0)), ((P_1; 1)) and ((\vec{P}_2; 0))</td>
</tr>
</tbody>
</table>

Table 1: Types of conics defined by Bézier curves with control mass points.

If \(f(\omega_2, \omega_1, \omega_0) = 7\), the control points are weighted points: the curve is an elliptical arc, a parabolic arc or a hyperbolic arc. If \((\omega_2, \omega_1, \omega_0) = (1, -1, 1)\), the parabolic arc is not bounded and for \(t = \frac{1}{2}\), the mass point is a direction vector of the parabola axis. If \((\omega_2, \omega_1, \omega_0) = (1, -2, 1)\), the hyperbolic arc is not bounded and there exists \(t\) in \([0, 1]\) such as the mass point is a direction vector of one of the asymptotes of the hyperbola.

If \(f(\omega_2, \omega_1, \omega_0) = 1\), the first control point is a weighted point, the others are vectors: the curve is a parabolic arc. The Bézier curve is defined by

\[
\begin{aligned}
&\frac{1}{\omega_0 B_0(t_0)} \left(\omega_0 B_0(t_0) \vec{OP}_0 + B_1(t_0) \vec{P}_1 + B_2(t_0) \vec{P}_2\right) & \text{if } t_0 \in [0, 1[ \\
&\frac{1}{\vec{P}_0} & \text{if } t_0 = 1
\end{aligned}
\]  
(5)

If \(f(\omega_2, \omega_1, \omega_0) = 4\), the Bézier curve can be defined in the same way.

If \(f(\omega_2, \omega_1, \omega_0) = 2\), the intermediate control point is a weighted point, the others are vectors: the curve is a branch of a hyperbola. The Bézier curve is defined by

\[
\begin{aligned}
&\frac{1}{\omega_1 B_1(t_0)} \left(\omega_1 B_1(t_0) \vec{OP}_1 + B_0(t_0) \vec{P}_0 + B_2(t_0) \vec{P}_2\right) & \text{if } t_0 \in ]0, 1[ \\
&\frac{1}{\vec{P}_0} & \text{if } t_0 = 0 \\
&\frac{1}{\vec{P}_2} & \text{if } t_0 = 1
\end{aligned}
\]  
(6)

and the centre of the hyperbola is \(P_1\). The vector \(\vec{P}_0\) is a direction vector of an asymptote of the hyperbola whereas the vector \(\vec{P}_2\) is a direction vector of the other asymptote.

If \(f(\omega_2, \omega_1, \omega_0) = 5\), the intermediate control point is a vector, the others are weighted points: the curve is an elliptical arc. The Bézier curve is defined by

\[
\begin{aligned}
&\frac{1}{\omega_0 B_0(t_0) + \omega_2 B_2(t_0)} \left(\omega_0 B_0(t_0) \vec{OP}_0 + B_1(t_0) \vec{P}_1 + \omega_2 B_2(t_0) \vec{OP}_2\right), \ t_0 \in [0, 1]
\end{aligned}
\]  
(7)

and the tangent vector to the curve at \(P_0\) or \(P_2\) is parallel to \(\vec{P}_1\).

### 4.2 Syntax

```latex
\psRQBCmasses[Options] (x_0, y_0)(x_1, y_1)(x_2, y_2)\{w_0, w_1, w_2\}
```

For the coordinates of the points all possible kinds of coordinates are possible, like polar, PostScript, nodes, ...
4.3 Three weighted orthogonal points

\begin{pspicture}[showgrid](-6,-6.4)(3,3)
\psframe(-6,-6)(3,3)
\psRQBCmasse[linecolor=blue](2,0)(2,2)(0,2){1,-1,1}
\psRQBCmasse[linecolor=navy,autoTrace](2,0)(2,2)(0,2){1,1,1}
\rput(P0){$P_0$} \uput[r](P1){$P_1$} \uput[r](P2){$P_2$
\end{pspicture}

4.4 Half-ellipse

\begin{pspicture}[showgrid](-3,-2.4)(3,2)
\psframe(-3,-2)(3,2)
\psRQBCmasse[linecolor=red,autoTrace](2,0)(0,1)(-2,0){1,0,1}
\rput[r](P0P1){$\overrightarrow{P_1}$} \uput[r](P1){$P_1$} \uput[r](P2){$P_2$
\rput(P1P2){$\overrightarrow{P_0}$} \uput[r](P0){$P_0$
\psRQBCmasse[linecolor=orange,autoTrace=false](2,0)(0,-1)(-2,0){1,0,1}
\end{pspicture}
### 4.5 Half-parabola

**Point** $P_2$ and two vectors

\begin{pspicture}[showgrid](-3,-3.4)(3,3)
\psclip{\psframe(-3,-3)(3,3)}
\psRQBCmasse[linecolor=red,autoTrace](2,0)(0,1)(-1,0){0,0,1}
\uput[r](P1P2){$\overrightarrow{P_1}$} \uput[r](P2){$P_2$}
\psRQBCmasse[linecolor=orange,autoTrace=false](2,0)(0,-1)(-1,0){1,0,0}
\uput[r](P0P2){$\overrightarrow{P_0}$}
\endpsclip
\end{pspicture}

**Point** $P_0$ and two vectors

\begin{pspicture}[showgrid](-3,-3.4)(3,3)
\psclip{\psframe(-3,-3)(3,3)}
\psRQBCmasse[linecolor=red,autoTrace](2,0)(0,1)(-1,0){1,0,0}
\uput[r](P0P1){$\overrightarrow{P_1}$} \uput[r](P0){$P_0$}
\psRQBCmasse[linecolor=orange,autoTrace=false](2,0)(0,-1)(-1,0){1,0,0}
\endpsclip
\end{pspicture}
4.6 Branch of a hyperbola

\begin{pspicture}[showgrid](-3,-3.4)(3,3)
\psclip{\psframe(-3,-3)(3,3)}
\psRQBCmasse[linecolor=red,autoTrace](1,1)(0,0)(-1,1){0,1,0}
\uput[r](0,0.5){$\overrightarrow{P_0}$}
\psRQBCmasse[linecolor=orange,autoTrace=false](1,1)(0,0)(-1,1){0,-1,0}
\uput[u](0,-0.5){$P_1$}
\endpsclip
\end{pspicture}

4.7 Parabola

\psset{unit=0.5}
\begin{pspicture}(-14,-3.4)(15,10)
\psclip{\psframe(-14,-3)(15,10)}
\psRQBCmasse[linecolor=red,autoTrace](0,6)(-13,0)(-1,-1){1,1,1}
\psRQBCmasse[linecolor=orange](0,6)(-13,0)(-1,-1){1,-1,1}
\uput[u](0,6){$P_0$}\uput[l](-13,0){$P_1$}\uput[d](-1,-1){$P_2$}
\endpsclip
\end{pspicture}
4.8 Ellipse

\psset{unit=0.5}
\begin{pspicture}(-14,-3.4)(15,10)
\psgrid[subgriddiv=0,gridcolor=lightgray,griddots=5,gridlabels=0pt]
\psplotImp[linewidth=0.5pt,linecolor=blue,algebraic](-6,-3)(15,10){-0.044*x^2-0.161*y^2 + 0.075*x*y + 0.074*x + 0.797*y + 1}
\psRQBCmasse[nPoints=20,autoTrace,showpoints](0,6)(-13,0)(-1,-1){1,0.5,1}
\psRQBCmasse[nPoints=40,linecolor=red,showpoints](0,6)(-13,0)(-1,-1){1,-0.5,1}
\psaxes[labelFontSize=\scriptscriptstyle]{->}(0,0)(-14,-3)(15,10)
\end{pspicture}

4.9 Complete circle

\psset{unit=1}
\begin{pspicture}(-4,-4.4)(4,4)
\psgrid[subgriddiv=0,gridcolor=lightgray,griddots=5,gridlabels=0pt]
\psRQBCmasse[autoTrace](0,3)(3,3)(3,0){1,1,2}
\psRQBCmasse[linecolor=red](0,3)(3,3)(3,0){1,-1,2}
\psaxes[labelFontSize=\scriptscriptstyle]{->}(0,0)(-4,-4)(4,4)
\end{pspicture}
We get a circle because we have

\[
\begin{align*}
\omega_0 \times \omega_2 \times P_0 P_2^2 &= 4 \times \overrightarrow{P_1'^2} \\
\overrightarrow{P_0 P_2} &\perp \overrightarrow{P_1'}
\end{align*}
\]

(8)
4.10 Animations

\(w_0 = 1, \ w_2 = 1 \text{ and a variable } w_1\)

With the beginning of \(w_1 = 0\) the curves are swapped. In the case of Bezier curves \(w_1 = 0\) gives only the \([P_0, P_2]\) segment. Using the mass points, the point \(P_1\) no longer exists but we get the vector \(\vec{P}_1\).
$w_0 = 1, |w_1| = 1$ and a variable $w_2$

The use of $|w_1|$ provides both arcs and the whole cone.

\begin{animateinline}[controls,loop, palindrome, begin={\begin{pspicture}(-8,-4)(4,4)},\end{pspicture}}]{3}\% 3 images/s
\multiframe{80}{rA=4.0+-0.1}{%
\psgrid[subgriddiv=0, gridcolor=lightgray, griddots=5, gridlabels=0pt]
\psclip{\psframe(-8,-4)(4,4)}
\psRQBCmasse[autoTrace,linewidth=1.5pt](0,-1)(1,0)(0,1){1,1,\rA}
\uput[u](P2){$P_2$}\uput[l](P1){$P_1$}\uput[d](P0){$P_0$}
\psRQBCmasse[linecolor=red,linewidth=1.5pt](0,-1)(1,0)(0,1){1,-1,\rA}
\psaxes[labelFontSize=\scriptscriptstyle,linewidth=0.01]{->}(0,0)(-8,-4)(4,4)
\rput{rb}(3.5,3){$w_2=\rA$}\%
\endpsclip}
\end{animateinline}
5 List of all optional arguments for \texttt{pst-bezier}

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<th>Type</th>
<th>Default</th>
</tr>
</thead>
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<td>ordinary</td>
<td>0.25</td>
</tr>
<tr>
<td>\texttt{nPoints}</td>
<td>ordinary</td>
<td>[none]</td>
</tr>
<tr>
<td>\texttt{showPolygon}</td>
<td>boolean</td>
<td>true</td>
</tr>
<tr>
<td>\texttt{autoTrace}</td>
<td>boolean</td>
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